Topic: Sum-difference identities

Question: Which of the following is equal to the sum?

$$(-\sin 70^\circ)(\cos 42^\circ) + (\cos 70^\circ)\sin(42^\circ)$$

Answer choices:

A $\cos 112^\circ$

B $\sin 28^\circ$

C $\sin(-28^\circ)$

D $\sin(-112^\circ)$

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Solution: C

Let $\theta = -70°$ and $\alpha = 42°$. By the sum identity for the sine function,

$$\sin(\theta + \alpha) = \sin\theta\cos\alpha + \cos\theta\sin\alpha$$

$$\sin(-70° + 42°) = (\sin -70°)(\cos 42°) + (\cos -70°)(\sin 42°)$$

By the odd identity $\sin\theta = -\sin(-\theta)$ and the even identity $\cos\theta = \cos(-\theta)$, this equation becomes

$$\sin(-70° + 42°) = (-\sin 70°)(\cos 42°) + (\cos 70°)(\sin 42°)$$

Now $\theta + \alpha = -70° + 42° = -28°$. Therefore,

$$\sin -28° = (-\sin 70°)(\cos 42°) + (\cos 70°)(\sin 42°)$$

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Question: Find the exact values.

Let θ be an angle in the second quadrant whose sine is $1/3$, and let α be an angle in the fourth quadrant whose cosine is $2/\sqrt{5}$. What are the exact values of $\sin(\theta + \alpha)$ and $\cos(\theta - \alpha)$?

Answer choices:

A $\sin(\theta + \alpha) = \dfrac{2\left(1 + \sqrt{2}\right)}{3\sqrt{5}}$ and $\cos(\theta - \alpha) = -\dfrac{\left(4\sqrt{2} + 1\right)}{3\sqrt{5}}$

B $\sin(\theta + \alpha) = \dfrac{2\left(1 - \sqrt{2}\right)}{3\sqrt{5}}$ and $\cos(\theta - \alpha) = -\dfrac{\left(4\sqrt{2} + 1\right)}{3\sqrt{5}}$

C $\sin(\theta + \alpha) = \dfrac{2\left(1 + \sqrt{2}\right)}{3\sqrt{5}}$ and $\cos(\theta - \alpha) = \dfrac{4\sqrt{2} - 1}{3\sqrt{5}}$

D $\sin(\theta + \alpha) = -\dfrac{2\left(1 + \sqrt{2}\right)}{3\sqrt{5}}$ and $\cos(\theta - \alpha) = \dfrac{4\sqrt{2} - 1}{3\sqrt{5}}$

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Solution: A

We'll apply sum-difference identities, so we first need to find $\cos\theta$ and $\sin\alpha$.

The basic Pythagorean identity tells us that

$$\sin^2\theta + \cos^2\theta = 1$$

Therefore,

$$\cos^2\theta = 1 - \sin^2\theta$$

Substituting $1/3$ for $\sin\theta$, we obtain

$$\cos^2\theta = 1 - \left(\frac{1}{3}\right)^2 = 1 - \frac{1}{9} = \frac{8}{9}$$

Since θ is in the second quadrant, we know that $\cos\theta$ is negative, so

$$\cos\theta = -\sqrt{\frac{8}{9}} = -\frac{\sqrt{8}}{\sqrt{9}} = -\frac{\sqrt{4(2)}}{3} = -\frac{2\sqrt{2}}{3}$$

Similarly, the basic Pythagorean identity tells us that

$$\sin^2\alpha + \cos^2\alpha = 1$$

Therefore,

$$\sin^2\alpha = 1 - \cos^2\alpha$$

Substituting $2/\sqrt{5}$ for $\cos\alpha$, we obtain

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$$\sin^2\alpha = 1 - \left(\frac{2}{\sqrt{5}}\right)^2 = 1 - \frac{4}{5} = \frac{1}{5}$$

Since α is in the fourth quadrant, we know that $\sin\alpha$ is negative, so

$$\sin\alpha = -\sqrt{\frac{1}{5}} = -\frac{1}{\sqrt{5}}$$

By the sum identity for the sine function,

$$\sin(\theta + \alpha) = \sin\theta\cos\alpha + \cos\theta\sin\alpha$$

$$\sin(\theta + \alpha) = \left(\frac{1}{3}\right)\left(\frac{2}{\sqrt{5}}\right) + \left(-\frac{2\sqrt{2}}{3}\right)\left(-\frac{1}{\sqrt{5}}\right)$$

$$\sin(\theta + \alpha) = \frac{2}{3\sqrt{5}} + \frac{2\sqrt{2}}{3\sqrt{5}}$$

$$\sin(\theta + \alpha) = \frac{2\left(1 + \sqrt{2}\right)}{3\sqrt{5}}$$

By the difference identity for the cosine function,

$$\cos(\theta - \alpha) = \cos\theta\cos\alpha + \sin\theta\sin\alpha$$

$$\cos(\theta - \alpha) = \left(-\frac{2\sqrt{2}}{3}\right)\left(\frac{2}{\sqrt{5}}\right) + \left(\frac{1}{3}\right)\left(-\frac{1}{\sqrt{5}}\right)$$

$$\cos(\theta - \alpha) = -\frac{4\sqrt{2}}{3\sqrt{5}} - \frac{1}{3\sqrt{5}}$$

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$$\cos(\theta - \alpha) = -\frac{\left(4\sqrt{2}+1\right)}{3\sqrt{5}}$$

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Topic: Sum-difference identities

Question: What are the exact values of $\tan(\theta + \alpha)$ and $\tan(\theta - \alpha)$ if θ is an angle in the second quadrant whose cosine is $-3/5$ and α is an angle in the third quadrant whose sine is $-2/\sqrt{5}$?

Answer choices:

A $\tan(\theta + \alpha) = \sqrt{\frac{11}{3}}$ and $\tan(\theta - \alpha) = 3$

B $\tan(\theta + \alpha) = \frac{2}{11}$ and $\tan(\theta - \alpha) = 2$

C $\tan(\theta + \alpha) = -\sqrt{\frac{3}{11}}$ and $\tan(\theta - \alpha) = -2$

D $\tan(\theta + \alpha) = \frac{11}{3}$ and $\tan(\theta - \alpha) = -3$

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Solution: B

We'll apply sum-difference identities, so we first need to find $\sin\theta$ and $\cos\alpha$.

The basic Pythagorean identity tells us that

$$\sin^2\theta + \cos^2\theta = 1$$

Therefore,

$$\sin^2\theta = 1 - \cos^2\theta$$

Substituting $-3/5$ for $\cos\theta$, we obtain

$$\sin^2\theta = 1 - \left(-\frac{3}{5}\right)^2 = 1 - \frac{9}{25} = \frac{16}{25}$$

Since θ is in the second quadrant, we know that $\sin\theta$ is positive, so

$$\sin\theta = \sqrt{\frac{16}{25}} = \frac{\sqrt{16}}{\sqrt{25}} = \frac{4}{5}$$

Similarly, the basic Pythagorean identity tells us that

$$\sin^2\alpha + \cos^2\alpha = 1$$

Therefore,

$$\cos^2\alpha = 1 - \sin^2\alpha$$

Substituting $-2/\sqrt{5}$ for $\sin\alpha$, we obtain

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$$\cos^2\alpha = 1 - \left(-\frac{2}{\sqrt{5}}\right)^2 = 1 - \frac{4}{5} = \frac{1}{5}$$

Since α is in the third quadrant, we know that $\cos\alpha$ is negative, so

$$\cos\alpha = -\sqrt{\frac{1}{5}} = -\frac{1}{\sqrt{5}}$$

Next, we'll find $\tan\theta$ and $\tan\alpha$.

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\left(\frac{4}{5}\right)}{\left(-\frac{3}{5}\right)} = -\frac{4}{3}$$

and

$$\tan\alpha = \frac{\sin\alpha}{\cos\alpha} = \frac{\left(-\frac{2}{\sqrt{5}}\right)}{\left(-\frac{1}{\sqrt{5}}\right)} = 2$$

By the sum identity for the tangent function,

$$\tan(\theta+\alpha) = \frac{\tan\theta + \tan\alpha}{1 - \tan\theta\tan\alpha}$$

$$\tan(\theta+\alpha) = \frac{\left(-\frac{4}{3} + 2\right)}{1 - \left(-\frac{4}{3}\right)(2)}$$

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$$\tan(\theta+\alpha)=\frac{\left(\frac{2}{3}\right)}{1+\left(\frac{8}{3}\right)}$$

$$\tan(\theta+\alpha)=\frac{\left(\frac{2}{3}\right)}{\left(\frac{11}{3}\right)}$$

$$\tan(\theta+\alpha)=\frac{2}{11}$$

By the difference identity for the tangent function,

$$\tan(\theta-\alpha)=\frac{\tan\theta-\tan\alpha}{1+\tan\theta\tan\alpha}$$

$$\tan(\theta-\alpha)=\frac{\left(-\frac{4}{3}-2\right)}{1+\left(-\frac{4}{3}\right)(2)}$$

$$\tan(\theta-\alpha)=\frac{\left(-\frac{10}{3}\right)}{1-\left(\frac{8}{3}\right)}$$

$$\tan(\theta-\alpha)=\frac{\left(-\frac{10}{3}\right)}{\left(-\frac{5}{3}\right)}$$

$$\tan(\theta-\alpha)=2$$

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